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DYNAMIC ASYMMETRIC LEVERAGE IN STOCHASTIC VOLATILITY MODELS

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□ *In the class of stochastic volatility (SV) models, leverage effects are typically specified through the direct correlation between the innovations in both returns and volatility, resulting in the dynamic leverage (DL) model. Recently, two asymmetric SV models based on threshold effects have been proposed in the literature. As such models consider only the sign of the previous return and neglect its magnitude, this paper proposes a dynamic asymmetric leverage (DAL) model that accommodates the direct correlation as well as the sign and magnitude of the threshold effects. A special case of the DAL model with zero direct correlation between the innovations is the asymmetric leverage (AL) model. The dynamic asymmetric leverage models are estimated by the Monte Carlo likelihood (MCL) method. Monte Carlo experiments are presented to examine the finite sample properties of the estimator. For a sample size of* $T = 2000$ *with 500 replications, the sample means, standard deviations, and root mean squared errors of the MCL estimators indicate only a small finite sample bias. The empirical estimates for S&P 500 and TOPIX financial returns, and USD/AUD and YEN/USD exchange rates, indicate that the DAL class, including the DL and AL models, is generally superior to threshold SV models with respect to AIC and BIC, with AL typically providing the best fit to the data.*

Keywords Asymmetric effects; Monte Carlo likelihood; Stochastic volatility; Threshold effects.

JEL Classification C220.

1. INTRODUCTION

It has long been recognized that the returns to financial assets are negatively correlated with changes in the volatilities of returns (see Black, 1976 and Christie, 1982), which leads to the "leverage" effect. Such an effect has been analyzed in the class of stochastic volatility

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(SV) models by Wiggins (1987), Chesney and Scott (1989), and Harvey and Shephard (1996), in which the specification is based on the direct (negative) correlation between the innovations in both returns and volatility. Although this direct correlation model is a specific type of asymmetric SV, we will refer to the asymmetric behavior based on the direct correlation between the pair of innovations as the dynamic leverage (DL) SV model to distinguish it from alternative specifications of asymmetric behavior that will be developed and discussed below.

Recently, So et al. (2002) and Asai and McAleer (2004) considered another type of asymmetry for SV models. So et al. (2002) proposed a threshold SV (TSV) model, which is analogous to the threshold ARCH model of Li and Li (1996). In the TSV model, the constant term and the autoregressive parameter in the SV equation changed according to the sign of the previous return. So et al. (2002) developed a Bayesian Markov chain Monte Carlo (MCMC) method to estimate the model. Asai and McAleer (2004) considered an asymmetric SV model using a threshold effects indicator function, as suggested in Glosten et al. (1992) (GJR) in the context of ARCH models. They derived the analytical relationship between the GJR model and the DL model and proposed a more general dynamic leverage threshold effects (DLTE) model. McAleer (2005) provides a comparison of various univariate and multivariate SV models that are available in the literature.

As So et al. (2002) and Asai and McAleer (2004) modeled only the sign of the previous return and neglected its magnitude, a more general dynamic asymmetric leverage SV model that accommodates both the sign and the magnitude of returns is proposed in this paper. We propose a dynamic asymmetric leverage (DAL) model that incorporates both the sign and the magnitude of previous returns in an SV model. A special case of the DAL model with zero direct correlation between the innovations is the asymmetric leverage (AL) model.

The remainder of the paper is organized as follows. Section 2 develops the AL and DAL models and briefly discusses two threshold SV models. Section 3 investigates the finite sample properties of the Monte Carlo likelihood (MCL) estimator, which is proposed by Sandmann and Koopman (1998), for the DAL models. In Section 4, variations of the DAL and TSV models are estimated using S&P 500 and Tokyo stock price index (TOPIX) financial returns, and exchange rates between the USA and Australia (USD/AUD) and Japan and the USA (YEN/USD). Section 5 discusses the differences between the DL and AL models in light of the correlation between the disturbances. Section 6 provides some concluding remarks.

2. DYNAMIC ASYMMETRIC EFFECTS IN STOCHASTIC VOLATILITY MODELS

Asymmetry is captured in the dynamic leverage (DL) model through the direct negative correlation between returns and volatility innovations:

$$y_t = \varepsilon_t \exp(h_t/2), \qquad t = 1, \ldots, T, \\ \varepsilon_t \sim N(0, 1), \tag{1}$$

$$h_{t+1} = \mu + \phi h_t + \eta_t, \\ \eta_t \sim N(0, \sigma_\eta^2), \\ E(\varepsilon_t \eta_t) = \rho \sigma_\eta, \tag{2}$$

where y_t is the mean-adjusted return on an asset and h_t denotes stochastic volatility. Asai and McAleer (2004) refer to this type of asymmetry, namely when $\rho < 0$, as the DL SV model. When $\rho = 0$, there is no dynamic leverage between the innovations to returns and volatility.

In this paper we propose a dynamic asymmetric leverage (DAL) model, in which volatility is affected by both the sign and the magnitude of the previous returns:

$$h_{t+1} = \mu + \phi h_t + \gamma_1 y_t + \gamma_2 |y_t| + \eta_t, \\ \eta_t \sim N(0, \sigma_\eta^2), \\ E(\varepsilon_t \eta_t) = \rho \sigma_\eta. \tag{3}$$

If $\gamma_1 = \gamma_2 = 0$, then the DAL model reduces to the standard DL model. A special case of the DAL model with zero direct correlation between the innovations is the asymmetric leverage (AL) model. When $\rho = 0$ and ε_t follows an AR(1) process, (3) is the model considered in Danielsson (1994) (although the asymmetric term, $\gamma_1 y_t$, was omitted in the empirical analysis). If the unobservable shocks, ε_t and $|\varepsilon_t|$, were included in (3) instead of the observable data, y_t and $|y_t|$, as in the exponential GARCH (EGARCH) model of Nelson (1991), estimation of the resulting SV model would be made far more computationally complicated.

In Equation (3) with $\rho = 0$, h_{t+1} is linear in y_t with slope given by $\gamma_1 + \gamma_2$ when y_t is positive, while h_{t+1} is linear in y_t with slope given by $\gamma_1 - \gamma_2$ when y_t is negative. Both the AL and the DAL model require $\gamma_1 + \gamma_2 < -(\gamma_1 - \gamma_2)$, so that $\gamma_1 < 0$, since the magnitude of the effect of a one-unit positive shock should be smaller than that of a one-unit negative shock. According to Black (1976), leverage may also be interpreted as requiring that $\gamma_1 + \gamma_2 < 0$ and $\gamma_1 - \gamma_2 < 0$, which implies $\gamma_1 < 0$ and $\gamma_1 < \gamma_2 < -\gamma_1$.

The direct application of the DAL model to real data may raise a problem for the following reason. Given ε_t, the log-volatility equation of the DAL model becomes

$$h_{t+1} = \mu + \phi h_t + \rho\sigma_\eta\varepsilon_t + \{\gamma_1\varepsilon_t + \gamma_2|\varepsilon_t|\}\exp(h_t/2) + \eta_t^*,$$

where $\eta_t^* \sim N\big(0, \sigma_\eta^2(1-\rho^2)\big)$. As shown in the appendix, the presence of ε_t and $\varepsilon_t \exp(h_t/2)$ lead to multicollinearity. In order to accommodate this effect, we propose an alternative DL model (DL2):

$$\begin{aligned} h_{t+1} &= \mu + \phi h_t + \gamma_2|y_t| + \eta_t, \\ \eta_t &\sim N(0, \sigma_\eta^2), \\ E(\varepsilon_t\eta_t) &= \rho\sigma_\eta, \end{aligned} \tag{4}$$

which is obtained by setting $\gamma_1 = 0$ in the DAL model.

The DL, AL, and DL2 models can be estimated by the Monte Carlo likelihood (MCL) method that was proposed by Sandmann and Koopman (1998) and Asai (2000). In this approach, the likelihood function based on $\log y_t^2$ can be approximated arbitrarily by decomposing it into a Gaussian component, which is constructed using the Kalman filter, and a remainder function, for which the expectation is evaluated through simulation. It should be noted that, after transforming y_t into $\log y_t^2$, the information regarding the correlation coefficient, ρ, is lost, but it can be recovered by conditioning on the signs of the original observations (for further details, see Harvey and Shephard, 1996). Based on Sandmann and Koopman (1998), Asai (2005) proposes to use a nearest covariance matrix in the Frobenius norm, in order to guarantee the positive definiteness of the importance sampling distribution of the MCL. The method is also used in the Monte Carlo simulations conducted in Asai and McAleer (2004). The finite sample properties of the MCL estimator will be investigated in the following section.

The smoothed volatility estimates of the DAL model, $\tilde{\sigma}_t$, which are estimates of $\exp(h_t/2)$, can be obtained by the method of Sandmann and Koopman (1998). As both ρ and γ_1 play key roles in describing leverage effects, we will calculate the correlation coefficients between the volatility increments, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t\tilde{\sigma}_t^{-1}$, in the empirical analysis to check for the negative correlation.

Asai and McAleer (2004) considered a special case of Equation (3),

$$\begin{aligned} h_{t+1} &= \mu + \phi h_t + \xi_t, \\ \xi_t &= \gamma\{I(\varepsilon_t) - E[I(\varepsilon_t)]\} + \eta_t, \end{aligned} \tag{5}$$

in which $I(\cdot)$ is an indicator function such that $I(x) = 1$ if $x < 0$ and $I(x) = 0$ otherwise. Unlike Equation (3), this threshold effects (TE) model captures the signs of the returns but neglects their magnitudes. The Monte Carlo results presented in Asai and McAleer (2004) indicated that the bias in the MCL estimator was small and that the coverage probability was close to the true value. The authors also presented empirical evidence that the TE model was generally less successful in capturing the leverage effects than the DL model.

So et al. (2002) recently proposed another type of threshold effects model,

$$h_{t+1} = \mu_{s_t} + \phi_{s_t} h_t + \eta_t, \tag{6}$$

in which s_t is a state variable such that

$$s_t = \begin{cases} 0 & \text{if } y_t < 0, \\ 1 & \text{otherwise.} \end{cases} \tag{7}$$

So et al. (2002) referred to this model as the threshold SV (TSV) model, in which ϕ_0 can be different from ϕ_1. In the symmetric case, the two sets of parameters are identical. If $\phi_0 = \phi_1$, then $\mu_0 \geq \mu_1$ implies that the variance is higher when the past return is negative than when it is positive. Indeed, ϕ_{s_t} measures the effect of the past variance on the current variance. If $\phi_0 > \phi_1$, the past variance will have a greater impact on the current variance after a fall in price than after an increase in price, as it would be expected to take a longer time for the bad news contained in the past variance to be digested by the market.

An estimation method based on the Bayesian MCMC method was proposed in So et al. (2002). However, upon taking logarithms of y_t^2, the TSV model can be shown to have a state space form with time-varying parameters, in which case the MCL method of Sandmann and Koopman (1998) can also be used to estimate the TSV model. For purposes of a direct comparison of the various threshold effects and dynamic leverage models, we will use the MCL method to estimate the various models to obtain the maximized log-likelihood values, the smoothed volatility, and the correlation coefficients between the increment in the smoothed volatility and the current standardized return.

3. MONTE CARLO EXPERIMENTS

In this section, we conduct some Monte Carlo experiments in order to investigate the finite sample property of the MCL estimators for the AL and DL2 models. As the benchmark of comparison, we use the performance for the DL model for the following reason. Regarding the simpler SV models, Sandmann and Koopman (1998) and Asai and McAleer (2004) provided some limited Monte Carlo experiments regarding the finite sample properties of the MCL estimator. Sandmann and Koopman (1998) performed Monte Carlo experiments for the basic SV model, that is, the SV model obtained by setting $\rho = 0$ in the DL model. They found that the sample means, standard deviations, and mean squared errors of the MCL estimator are usually close to those of the Bayesian MCMC, as obtained in Jacquier et al. (1994). Asai and McAleer (2004) conducted Monte Carlo experiments for three types of asymmetric SV model, that is, the DL and TE models as well as a more general model that nests both SV models. Their empirical results also show that the biases of the MCL estimator are generally small, and that the 95% coverage probabilities are close to the true values. The simulation experiments in this paper are conducted to minimize overlap with the existing Monte Carlo results.

As it is not easy to derive an exact relation among the DL, AL, and DL2 models, we consider a relation among the approximations in order to specify the parameter values for the experiments (see the appendix for the details of the derivations). The parameter values are specified as

$$(\phi, \sigma_\eta, \mu, \rho, \gamma_1, \gamma_2) = \begin{cases} (0.95, 0.260, 0, -0.3, 0, 0) & \text{for DL,} \\ (0.93, 0.246, -0.04, 0, -0.078, 0.05) & \text{for AL,} \\ (0.93, 0.258, -0.04, -0.3, 0, 0.05) & \text{for DL2,} \end{cases}$$

in the framework of the DAL model. Roughly speaking, under these parameters, the three models have similar properties. For example, volatility persistence is approximately 0.95 for all the models.

It should be stressed that the relationship is based on the approximations. As the volatility process of the AL model is highly nonlinear, even if we were to consider a stationary process for the linear approximation, some of the simulated AL processes might turn out to be explosive. Since such simulated data are unfavorable for Monte Carlo simulations, we ensure that the volatility persistence is relatively low. For the parameter sets, only 0.9% of the generated AL processes were explosive. If we consider a negative value for γ_2, say $\gamma_2 = -0.05$, then the value of ϕ should be 0.97, which reflects the relationship.

We consider a sample size of $T = 2000$ with 500 replications. Table 1 shows the sample means, standard deviations, and root mean squared errors of the MCL estimators. The sample means are close to the true values for all models, indicating little bias. Compared with the Monte Carlo results in Table 3 of Sandmann and Koopman (1998), which is limited to the parameter values given by $\rho = \gamma_1 = \gamma_2 = 0$, the standard deviations and root mean squared errors presented in Table 1 seem quite reasonable.

TABLE 1 Finite sample performance of the MCL estimator for $T = 2000$

Parameters	DL	AL	DL2
ϕ	0.9444	0.9302	0.9275
	(0.0866)	(0.0219)	(0.0213)
	[0.0867]	[0.0219]	[0.0215]
σ_η	0.2569	0.2370	0.2542
	(0.0739)	(0.0268)	(0.0268)
	[0.0740]	[0.0283]	[0.0270]
μ	−0.0004	−0.0352	−0.0379
	(0.0072)	(0.0276)	(0.0271)
	[0.0072]	[0.0278]	[0.0271]
ρ	−0.2926		−0.2944
	(0.0752)		(0.0684)
	[0.0755]		[0.0687]
γ_1		−0.0788	
		(0.0150)	
		[0.0151]	
γ_2		0.0440	0.0476
		(0.0316)	(0.0306)
		[0.0322]	[0.0307]

Standard errors are in parentheses and root mean squared errors are in brackets.

4. EMPIRICAL RESULTS

This section examines the MCL estimates of asymmetric behavior in the SV, DL, AL, DAL, and TSV models for four sets of empirical data, namely Standard and Poor's 500 Composite Index (S&P 500), the Tokyo stock price index (TOPIX), the US Dollar/Australian Dollar exchange rate (USD/AUD), and the Japanese Yen/US dollar exchange rate (YEN/USD). The sample periods are given as follows: S&P 500, 1/6/1986 to 12/4/2000, giving $T = 3604$ observations; TOPIX, 1/4/1990 to 9/30/1999, giving $T = 2404$ observations; USD/AUD, 1/6/1986 to 11/15/2002, giving $T = 4123$ observations; and Yen/USD, 1/4/1990 to 12/28/1999, giving $T = 2467$ observations. The returns, R_t, are defined as $\Delta \log P_t = \log P_t - \log P_{t-1}$ times 100, where P_t is the closing price on day t.

Table 2 shows some descriptive statistics of R_t. Since the Ljung and Box (1978) test over-rejects the null hypothesis of no autocorrelation in the presence of heteroskedasticity, we use the test proposed by Wooldridge (1991), which is the robust LM (RLM) test for serial correlation in the presence of ARCH(m). The value $m = 2$ is used as the Monte Carlo simulations conducted by Silvapulle and Evans (1998). Asai (2000) show that the RLM tests assuming ARCH(2) have the correct size and sufficient power, even if the underlying processes are the GARCH(1,1) and the basic

TABLE 2 Descriptive statistics for returns and threshold estimates

Descriptive statistics	S&P Raw data	S&P TAR(1) residuals	TOPIX Raw data	TOPIX TAR(2) residuals	AUD/USD Raw data	YEN/USD Raw data
T	3604	3603	2404	2402	4123	2467
Mean	0.0546	0.0000	−0.0270	−0.0000	−0.0000	0.0000
Std. dev.	1.0243	1.0230	1.1290	1.1334	0.65917	0.8599
RLM(10)	22.033	11.607	45.023	13.694	14.155	9.910
	[0.015]	[0.312]	[0.000]	[0.187]	[0.166]	[0.448]
$Q^2(10)$	230.98	193.32	404.28	374.69	251.08	436.24
	[0.001]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Parameters						
c_0		0.0387		−0.0230		
		(0.0475)		(0.0571)		
c_1		−0.0474		−0.1801		
		(0.0611)		(0.0592)		
$\psi_{1,0}$		0.0862		0.1235		
		(0.0709)		(0.0553)		
$\psi_{1,1}$		−0.0729		0.0292		
		(0.0996)		(0.0637)		
$\psi_{2,0}$				−0.0558		
				(0.0397)		
$\psi_{2,1}$				−0.1357		
				(0.0589)		

RLM(·) is the autocorrelation test proposed by Wooldridge (1991), which is the robust LM test for serial correlation in the presence of ARCH(2). RLM(10) is based on ten lags and is applied to raw returns, R_t, or mean subtracted returns, $R_t - m_t$. $Q^2(10)$ denotes the portmanteau test, proposed by Ljung and Box (1978), for serial correlations based on ten lags about R_t^2 or $(R_t - m_t)^2$. The probability values are given in brackets. The second half of the table presents estimates of the parameters of $m_t = c_{s_{t-i}} + \sum_{i=1}^{p} \psi_{s_{t-i}} R_{t-i}$, where s_t is 0 if $y_t > 0$ and 1 otherwise. The heteroskedasticity robust standard errors of White (1980) are given in parentheses.

SV model. The RLM tests based on ten lags (RLM(10)) indicate that returns of S&P and TOPIX are serially correlated but that the exchange rate returns are serially uncorrelated.

The autocorrelation structure in the stock returns, $R_t - m_t$, was removed by using the following threshold AR (TAR) model:

$$m_t = c_{s_{t-i}} + \sum_{i=1}^{p} \psi_{s_{t-i}} R_{t-i},$$

where s_t is 0 if $y_t > 0$, and 1 otherwise. As for the returns minus the estimates of the conditional means, the RLM shows no evidence of autocorrelation in the presence of heteroskedasticity. The second half of Table 2 reports the estimates of the parameters of the TAR model, with the heteroskedasticity-consistent standard errors of White (1980) given in parentheses. The Ljung–Box tests for squared returns, R_t^2 and $(R_t - m_t)^2$, show strong evidence of autocorrelation in the volatility. Hereafter, for convenience we will refer to the stock and exchange rate returns series as $y_t = R_t - m_t$ and $y_t = R_t - \overline{R}$, respectively.

For stock returns such as S&P 500 and TOPIX, a negative correlation would be expected between the innovations in returns and volatility. Table 3 shows the MCL estimates for S&P 500 returns. Four kinds of DAL model were estimated, namely the standard SV model with $\rho = \gamma_1 = \gamma_2 = 0$, the DL with $\gamma_1 = \gamma_2 = 0$, the AL with $\rho = 0$, and the DL2 with $\gamma_1 = 0$. The significance of the estimates of ρ, γ_1, and/or γ_2 in the various models leads to a strong rejection of the basic SV model, which neglects the asymmetric effects. The estimates of γ_2 in the AL and DL2 model are negative and significant, indicating that the estimates of ϕ for these modes are larger than that of the DL model, as stated in the previous section. Although the estimate of ρ is significant and has the expected negative sign at the 5% level in the DL model, the LR test based on the DL2 model rejects the DL model. The estimates of ρ and γ_1 are significant, and have the expected negative signs, in the DL2 and AL models, respectively. The AIC and BIC choose the AL model among the four models. The correlation coefficient between the increment in the smoothed volatility, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t\tilde{\sigma}_t^{-1}$, is -0.3751 in the AL model, and even more strongly negative in the DL and DL2 models, while it is close to zero in the basic SV model.

The estimates using S&P 500 returns for the TSV model are also given in Table 3. It is clear that the estimates of ϕ_0 and μ_0 are larger than those

TABLE 3 MCL estimates of various DAL and TSV models for S&P 500 returns

Parameters	SV	DL	AL	DL2	Parameters	TSV
ϕ	0.9729	0.9563	0.9803	0.9674	ϕ_0	0.9959
	(0.0065)	(0.0090)	(0.0114)	(0.0121)		(0.0056)
σ_η	0.1860	0.2288	0.1949	0.2313	ϕ_1	0.9359
	(0.0199)	(0.0244)	(0.0226)	(0.0269)		(0.0147)
μ	−0.0115	−0.0195	0.0345	0.0073	σ_η	0.1955
	(0.0043)	(0.0057)	(0.0162)	(0.0148)		(0.0201)
ρ		−0.3424		−0.3587	μ_0	0.0674
		(0.0514)		(0.0510)		(0.0169)
γ_1			−0.1177		μ_1	−0.0948
			(0.0160)			(0.0190)
γ_2			−0.0636	−0.0327		
			(0.0176)	(0.0151)		
LogLike	−7874.6	−7858.2	−7839.5	−7855.0	LogLike	−7861.7
AIC	15755.1	15724.3	15689.9	15720.0	AIC	15733.4
BIC	15773.7	15749.1	15719.9	15751.0	BIC	15764.3
Corr	−0.0449	−0.5510	−0.3751	−0.6076	Corr	−0.4551

Standard errors are in parentheses. Corr denotes the correlation coefficient between the increment in smoothed volatility, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t\tilde{\sigma}_t^{-1}$.

of ϕ_1 and μ_1, respectively, which shows the existence of leverage effects, as discussed in Section 2. The correlation coefficient between the increment in the smoothed volatility and the current standardized return is -0.4551, which lies between the values for the DL and AL models. Values of AIC and BIC for the TSV model are smaller than those of the basic SV model but larger than those for the DL, AL, and DL2 models, which suggests that the TSV model is inadequate for capturing the asymmetric structure in stock returns in comparison with the performance of the DL, AL, and DAL models. Overall, the AL model fits the data best in terms of minimizing AIC and BIC.

Table 4 for TOPIX returns shows that the estimates of ρ are negative and significant in the DL and DL2 models but that the estimate of γ_2 is insignificant, indicating the rejection of the DL2 model. Both AIC and BIC suggest that AL is the best model. The correlation coefficient between the increment in the smoothed volatility and the current standardized return for AL is -0.6386. As in the case of S&P 500 returns, the standard SV model is clearly rejected in favor of the three SV models with leverage effects. In the TSV model, the estimates of ϕ_0 and μ_0 are larger than those of ϕ_1 and μ_1, respectively, which shows the existence of leverage effects. However, on the basis of AIC and BIC, the TSV model does not capture the asymmetric structure in stock returns in comparison with the DL, AL, and DAL models. Overall, in both Tables 1 and 2, there is clear and significant evidence of asymmetries in the DL, AL, DL2, and TSV models.

Tables 5 and 6 present the MCL estimates for the USD/AUD and Yen/USD returns, respectively. In Table 5, the results generally lead to

TABLE 4 MCL estimates of various DAL and TSV models for TOPIX returns

Parameters	SV	DL	AL	DL2	Parameters	TSV
ϕ	0.9526	0.9500	0.9780	0.9589	ϕ_0	0.9876
	(0.0106)	(0.0107)	(0.0188)	(0.0178)		(0.0144)
σ_η	0.2518	0.2622	0.1963	0.2596	ϕ_1	0.9143
	(0.0261)	(0.0285)	(0.0283)	(0.0289)		(0.0221)
μ	0.0042	0.0034	0.0371	0.0169	σ_η	0.2259
	(0.0054)	(0.0050)	(0.0245)	(0.0225)		(0.0247)
ρ		−0.4656		−0.4667	μ_0	0.1109
		(0.0490)		(0.0486)		(0.0186)
γ_1			−0.1050		μ_1	−0.1044
			(0.0127)			(0.0186)
γ_2			−0.0392	−0.0386		
			(0.0283)	(0.0224)		
LogLike	−5181.1	−5148.9	−5143.6	−5148.7	LogLike	−5158.5
AIC	10368.1	10305.8	10297.2	10307.5	AIC	10327.0
BIC	10385.4	10329.0	10326.2	10336.4	BIC	10355.9
Corr	−0.1463	−0.7774	−0.6386	−0.7673	Corr	−0.6377

Standard errors are in parentheses. Corr denotes the correlation coefficient between the increment in smoothed volatility, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t \tilde{\sigma}_t^{-1}$.

TABLE 5 MCL estimates of various DAL and TSV models for USD/AUD returns

Parameters	SV	DL	AL	DL2	Parameters	TSV
ϕ	0.9367	0.9288	0.9853	0.9856	ϕ_0	0.9445
	(0.0134)	(0.0149)	(0.0155)	(0.0160)		(0.0291)
σ_η	0.2566	0.2688	0.2321	0.2365	ϕ_1	0.9004
	(0.0285)	(0.0304)	(0.0266)	(0.0276)		(0.0278)
μ	−0.0719	−0.0812	0.0659	0.0653	σ_η	0.2838
	(0.0162)	(0.0180)	(0.0347)	(0.0354)		(0.0349)
ρ		−0.1294		−0.1052	μ_0	−0.0213
		(0.0504)		(0.0357)		(0.0349)
γ_1			−0.0681		μ_1	−0.1528
			(0.0208)			(0.0385)
γ_2			−0.1745	−0.1729		
			(0.0410)	(0.0417)		
LogLike	−8775.2	−8772.0	−8758.4	−8761.8	LogLike	−8769.6
AIC	17556.3	17552.0	17526.7	17533.6	AIC	17549.2
BIC	17575.3	17577.3	17558.3	17565.2	BIC	17580.8
Corr	−0.0091	−0.3396	−0.2368	−0.3245	Corr	−0.2970

Standard errors are in parentheses. Corr denotes the correlation coefficient between the increment in smoothed volatility, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t\tilde{\sigma}_t^{-1}$.

similar implications as in the case of the S&P 500 returns. The correlation coefficient between the increment in the smoothed volatility and the current standardized return is −0.2368 for AL. Both the DL and DL2 models have similar negative correlation coefficients, whereas the basic SV model again has a correlation coefficient that is very close to zero. In the

TABLE 6 MCL estimates of various DAL and TSV models for YEN/USD returns

Parameters	SV	DL	AL	DL2	Parameters	TSV
ϕ	0.9509	0.9349	0.9701	0.9468	ϕ_0	0.9933
	(0.0143)	(0.0184)	(0.0288)	(0.0296)		(0.0099)
σ_η	0.2362	0.2631	0.2342	0.2606	ϕ_1	0.8986
	(0.0331)	(0.0388)	(0.0418)	(0.0452)		(0.0268)
μ	−0.0461	−0.0606	0.0160	−0.0304	σ_η	0.2387
	(0.0145)	(0.0186)	(0.0532)	(0.0494)		(0.0329)
ρ		−0.1809		−0.1800	μ_0	0.0198
		(0.0594)		(0.0678)		(0.0206)
γ_1			−0.0767		μ_1	−0.1196
			(0.0235)			(0.0355)
γ_2			−0.0826	−0.0363		
			(0.0531)	(0.0470)		
LogLike	−5311.7	−5307.7	−5304.6	−5307.2	LogLike	−5307.4
AIC	10629.5	10623.4	10619.2	10624.4	AIC	10624.9
BIC	10647.0	10646.7	10648.2	10653.5	BIC	10653.9
Corr	−0.0303	−0.4799	−0.4417	−0.4350	Corr	−0.3156

Standard errors are in parentheses. Corr denotes the correlation coefficient between the increment in smoothed volatility, $\tilde{\sigma}_{t+1} - \tilde{\sigma}_t$, and the current standardized return, $y_t\tilde{\sigma}_t^{-1}$.

TSV model, the estimates of ϕ_0 and μ_0 are again larger than those of ϕ_1 and μ_1, respectively, which indicates the presence of leverage effects. Overall, AIC and BIC prefer the AL model, and the AL and DL2 models are superior to the DL and TSV models with regard to AIC and BIC.

The results for the YEN/USD returns in Table 6 are similar to those for TOPIX returns in Table 4. In both Tables 4 and 6, there is significant evidence of asymmetries in the DL, AL, DAL, and TSV models. Although the estimates of ρ are negative and significant in the DL and DL2 models, the estimate of γ_2 is insignificant, favoring the DL model. The estimate of γ_1 is negative and significant in the AL model, and both AIC and BIC suggest that AL is the best model. The correlation coefficient between the increment in the smoothed volatility and the current standardized return for AL is -0.4417, which is very close to the value for the DL2 model. In the TSV model, the estimates of ϕ_0 and μ_0 are again larger than those of ϕ_1 and μ_1, respectively, which indicates the existence of leverage effects.

5. DISCUSSION

This section discusses the differences between the DL and AL models in light of the correlation between the disturbances. In what follows, attention is focused on the AL model with the restriction $\gamma_2 = 0$ for the following two reasons. First, as stated in Section 2, the sign of γ_2 has no effects on the asymmetric structure, while the effect of its magnitude can be absorbed by the value of γ_1. The sign and magnitude of γ_2 clearly affect volatility persistence. Second, the empirical results indicate that some of the estimates of γ_2 for the AL model are insignificant.

Consider denoting a general disturbance in the volatility equation as $\xi_t = \gamma_1 \varepsilon_t e^{h_t/2} + \eta_t$. Then, given h_t, the mean and variance of ξ_t are zero and $\gamma_1^2 e^{h_t} + \sigma_\eta^2$, respectively. Furthermore, the correlation coefficient between ε_t and ξ_t, given h_t, is given by $\rho_t = \gamma_1 / \sqrt{\gamma_1^2 + \sigma_\eta^2 e^{-h_t}}$. If γ_1 is negative, then ρ_t is the monotonically decreasing function of h_t, and hence

$$\frac{\gamma_1}{\sqrt{\gamma_1^2 + \sigma_\eta^2}} < \rho_t < 0.$$

In other words, when volatility increases, the negative correlation becomes stronger. Overall, the AL model with the restriction $\gamma_2 = 0$ has another form:

$$y_t = \varepsilon_t e^{h_t/2},$$

$$\varepsilon_t \sim N(0, 1),$$

$$h_{t+1} = \mu + \phi h_t + \xi_t,$$

$$\xi_t \mid h_t \sim N(0, \gamma_1^2 e^{h_t} + \sigma_\eta^2),$$

$$\mathrm{Corr}(\varepsilon_t, \xi_t \mid h_t) = \rho_t = \gamma_1 / \sqrt{\gamma_1^2 + \sigma_\eta^2 e^{-h_t}}.$$

While the DL model has a constant correlation between the disturbances, the correlation coefficient for the AL model depends on the value of h_t. Therefore adding $\gamma_1 y_t$ to the volatility equation in the standard SV model makes the volatility structure possible, and not only to capture the leverage effect, but also to describe the time-varying correlation structure.

It is also possible to consider various time-varying correlations for the DL model directly, but this is a subject for future research.

6. CONCLUSION

In the class of stochastic volatility (SV) models, leverage effects are typically specified through the direct correlation between the innovations in both returns and volatility. This paper considered three methods for modeling asymmetries in SV models, namely the standard dynamic leverage (DL) model based on the negative correlation between the innovations in returns and volatility, the threshold effects SV (TSV) model proposed by So et al. (2002), in which the constant term and the autoregressive parameter in the volatility equation change according to the sign of the previous return, and a new dynamic asymmetric leverage (DAL) model based on both the direct negative correlation and also the sign and magnitude of the threshold effects. Two special cases of the DAL model were also proposed and discussed. One is the asymmetric leverage (AL) model, which has zero direct correlation between the innovations, and the other is an alternative DL (or DL2) model, which restricts the asymmetric parameter γ_1 to zero.

The dynamic asymmetric leverage models were estimated by the Monte Carlo likelihood (MCL) method. Monte Carlo experiments were presented to examine the finite sample properties of the estimator. For a sample size of $T = 2000$ with 500 replications, the sample means, standard deviations, and root mean squared errors of the MCL estimators were shown to be close to the true values for all models, thereby indicating only a small finite sample bias.

When these SV models were estimated using returns on four financial time series, the asymmetric effects were found to be statistically significant in each case. The empirical results for S&P 500 and TOPIX financial returns, and USD/AUD and YEN/USD exchange rates, indicated that the basic SV model, which ignored the asymmetric effects, was inadequate. The estimated correlation coefficients between the increment in the smoothed volatility and the current standardized return were always negative for the DL, AL, DL2, and TSV models. Models that accommodated dynamic and/or asymmetric leverage effects, namely DL, AL, and DAL, were superior to their threshold effects counterparts with respect to both AIC and BIC, with AL always providing the best fit to the data. Therefore the asymmetric leverage specification was always preferred empirically to its dynamic leverage counterpart.

APPENDIX: LINEAR APPROXIMATION OF THE DAL MODEL AND PARAMETER VALUES FOR MONTE CARLO SIMULATION

From the definition $\xi_t = \gamma_1 \varepsilon_t + \gamma_2 |\varepsilon_t|$, it follows that

$$E(\xi_1) = \gamma_2 \sqrt{2/\pi},$$
$$V(\xi_t) = \gamma_1^2 + \gamma_2^2 (1 - 2/\pi).$$

As the first-order Taylor approximation of $e^{h/2}\xi$ around $(h, \xi) = (a, b)$ is given by $e^{h/2}\xi \simeq e^{a/2}\xi + 0.5be^{a/2}(h - a)$, the volatility process of the DAL model can be approximated by

$$h_{t+1} = \left[\mu - \frac{\gamma_2}{\sqrt{2\pi}} a e^{a/2}\right] + \left[\phi + \frac{\gamma_2}{\sqrt{2\pi}} e^{a/2}\right] h_t + [(\rho\sigma_\eta + \gamma_1 e^{a/2})\varepsilon_t + (\gamma_2|\varepsilon_t| + \eta_t)], \quad \text{(A1)}$$

in which $b = \gamma_2\sqrt{2/\pi}$. By the approximation in (A1), it is clear that the presence of $\rho\sigma_\eta\varepsilon_t$ and $\gamma_1 y_t (=\gamma_1 \varepsilon_t e^{h_t/2})$ lead to multicollinearity. The value of a should be the unconditional mean of h_t.

Based on this approximation, we can determine the parameter values for the Monte Carlo experiments. Based on the empirical results of the DL model examined in Asai and McAleer (2004), we set the unconditional mean of h_t to zero. Since $a = 0$, we have $\mu = -\gamma_2\sqrt{2/\pi}$ for the DL and DL2 models. We also specified the second set of brackets in (A1) as 0.92, and the standard deviation of the third set of brackets as 0.260.

For the DL model, we set $\gamma_1 = \gamma_2 = 0$ and $\rho = -0.3$. The value of $\rho = -0.3$ is also used in the experiments of Harvey and Shephard (1996) and Asai and McAleer (2004). Hence the parameter values for the Monte Carlo experiments are given as

$$(\phi, \sigma_\eta, \mu, \rho) = (0.95, 0.260, 0, -0.3).$$

Second, the AL model needs $\rho = 0$. If we set $\gamma_2 = 0.05$, then we obtain

$$(\phi, \sigma_\eta, \mu, \gamma_1, \gamma_2) = (0.93, 0.246, -0.04, -0.078, 0.05).$$

Third, the DL2 model has $\gamma_1 = 0$. Under $\rho = -0.3$ and $\gamma_2 = 0.05$, it follows that

$$(\phi, \sigma_\eta, \mu, \rho, \gamma_2) = (0.93, 0.258, -0.04, -0.3, 0.05).$$

These are the three sets of parameter values that are used in the Monte Carlo simulations.

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REFERENCES

Asai, M. (2000). Testing for serial correlation in the presence of stochastic volatility. *Asia-Pacific Financial Markets* 7:321–337.

Asai, M. (2005). Autoregressive stochastic volatility models with heavy-tailed distribution: a comparison with multi-factor volatility models. Unpublished manuscript, Faculty of Economics, Tokyo Metropolitan University.

Asai, M., McAleer, M. (2004). Dynamic leverage and threshold effects in stochastic volatility models. Unpublished manuscript, Faculty of Economics, Tokyo Metropolitan University.

Black, F. (1976). Studies of stock market volatility changes. In: *1976 Proceedings of the American Statistical Association Business and Economic Statistics Section*. American Statistical Association, pp. 177–181.

Chesney, M., Scott, L. O. (1989). Pricing European currency options: a comparison of the modified black-scholes model and a random variance model. *Journal of Financial and Quantitative Analysis* 24:267–284.

Christie, A. A. (1982). The stochastic behavior of common stock variances: value, leverage and interest rate effects. *Journal of Financial Economics* 10:407–432.

Danielsson, J. (1994). Stochastic volatility in asset prices: estimation with simulated maximum likelihood. *Journal of Econometrics* 64:375–400.

Glosten, L., Jagannathan, R., Runkle, D. (1992). On the relation between the expected value and volatility of nominal excess returns on stocks. *Journal of Finance* 46:1779–1801.

Harvey, A. C., Shephard, N. (1996). Estimation of an asymmetric stochastic volatility model for asset returns. *Journal of Business and Economic Statistics* 14:429–434.

Hentschel, L. (1995). All in the family: nesting symmetric and asymmetric GARCH models. *Journal of Financial Economics* 39:71–104.

Jacquier, E., Polson, N. G., Rossi, P. E. (1994). Bayesian analysis of stochastic volatility models. *Journal of Business and Economic Statistics* 12:371–389.

Li, C. W., Li, W. K. (1996). On a double-threshold autoregressive heteroscedastic time series model. *Journal of Applied Econometrics* 11:253–274.

Ljung, G. M., Box, G. E. P. (1978). On a measure of lack of fit in time series models. *Biometrika* 65:297–303.

McAleer, M. (2005). Automated inference and learning in modeling financial volatility. *Econometric Theory* 21:232–261.

Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: a new approach. *Econometrica* 59:347–370.

Sandmann, G., Koopman, S. J. (1998). Estimation of stochastic volatility models via Monte Carlo maximum likelihood. *Journal of Econometrics* 87:271–301.

Silvapulle, P., Evans, M. (1998). Testing for serial correlation in the presence of dynamic heteroskedasticity. *Econometric Reviews* 17:31–55.

So, M. K. P., Li, W. K., Lam, K. (2002). A threshold stochastic volatility model. *Journal of Forecasting* 21:473–500.

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* 48:817–838.

Wiggins, J. B. (1987). Option values under stochastic volatility: theory and empirical estimates. *Journal of Financial Economics* 19:351–372.

Wooldridge, J. M. (1991). On the applications of robust, regression-based diagnostics to models of conditional means and conditional variances. *Journal of Econometrics* 47:5–46.