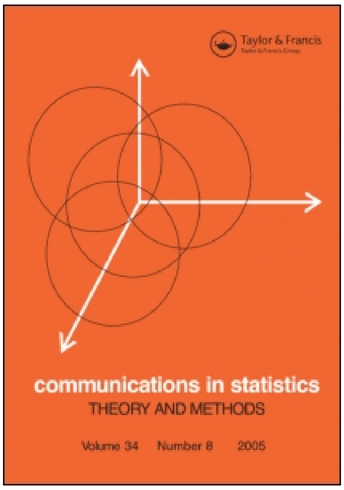

Communications in Statistics - Theory and Methods
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713597238

Some improved ratio type estimators of population mean and ratio in finite population sample surveys

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To cite this Article Prasad, B.(1989) 'Some improved ratio type estimators of population mean and ratio in finite population sample surveys', Communications in Statistics - Theory and Methods, 18: 1, 379 — 392
To link to this Article: DOI: 10.1080/03610928908829905
URL: http://dx.doi.org/10.1080/03610928908829905

COMMUN. STATIST.-THEORY METH. 18(1), 379-392 (1989)

SOME IMPROVED RATIO TYPE ESTIMATORS OF POPULATION MEAN AND RATIO IN FINITE POPULATION SAMPLE SURVEYS

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Key words and phrases: *population mean, population ratio, auxilary variate, prior information, ratio estimator, ratio type estimators.*

ABSTRACT

In this paper we present a class of ratio type estimators of the population mean and ratio in a finite population sample surveys with without replacement simple random sampling design, where information on an auxiliary variate x positively correlated with the main variate y is available. Large sample approximations to mean square errors (MSE) of these estimators are evaluated and their MSE's are compared with the MSE of the usual ratio estimator $\bar{y}_R$ of $\bar{Y}$ the population mean of y. It is shown that under certain conditions these estimators are more efficient than $\bar{y}_R$. When a prior knowledge of the value of the coefficient of variation, c_y, of y is at hand, ratio type estimator, say $\bar{y}_1$, of $\bar{Y}$ is proposed. It is shown, under certain conditions, that $\bar{y}_1$ is more efficient than $\bar{y}_R$. When values of c_y, c_x and the population correlation coefficient ρ is at hand, then we have proposed another estimator, say $\bar{y}_2$ of $\bar{Y}$, which is always better than $\bar{y}_R$ as far as the efficiency is concerned. In

fact, $\bar{y}_2$ is shown to be even better than $\bar{y}_1$. Finally estimators better than the usual ratio estimator $\bar{y}/\bar{x}$ of $\bar{Y}/\bar{X}$ are given.

1. INTRODUCTION

One of the significant developments in sample surveys over the last three decades is the use of information on an auxiliary character, correlated with the character under study, for obtaining estimates of the population total or population mean for the character under study. When such supplementary information is available, various estimators for the population total or mean have been devised for different sampling designs by using different methods of estimation. For example, when the sampling design is the simple random sampling and supplementary information on an auxiliary character correlated with the main character is available, the ratio and regression methods of estimation of the population mean are among the most widely adopted methods in large scale sample surveys since under certain circumstances, usually met with, such estimators are more efficient than the conventional unbiased estimators, such as mean per unit estimator.

In this paper we propose some ratio type estimators of the population mean $\bar{Y}$ (or population total Y) of a characteristic of interest y. In the ratio method an auxiliary variate x, correlated with y is measured for each unit in the sample. The population total X of x is assumed to be known. In practice, x may be the value of y at some previous occasion when a complete census or a sample survey was taken or it may concurrently be measured along with y on each unit in the sample, (e.g. weight x of an orange in the sample may be measured while measuring the content y of vitamin C in the orange). The aim in this method is to obtain increased precision by taking advantage of the correlation between y and x.

When the design adopted is simple random sampling the ratio estimate of the population mean $\bar{Y}$ is based on the sample means $\bar{y}$ and $\bar{x}$ of y and x. Thus the efficiency of the ratio estimate is very much dependent on the efficiency of $\bar{y}$ and $\bar{x}$ as estimators of $\bar{Y}$ and $\bar{X}$. In this paper, we propose some new ratio type estimators of $\bar{Y}$ based on some improved estimators $\bar{y}^*$ of $\bar{Y}$ and/or some improved estimators of $(\bar{Y}/\bar{X})$. We will show that under certain circumstances these ratio type estimators are more efficient than the usual ratio esitmator $(\bar{y}/\bar{x})\bar{X}$ of $\bar{Y}$.

In the sequel we will also present some estimators for the ratio of the population total for y to population total for x. These estimators are shown to be more efficient than the usual ratio estimator $(\bar{y}/\bar{x})$.

Throughout this paper we assume that the population under study is finite. The results for infinite population can be deduced directly from the results given here. We also assume throughout that the sampling design adopted is simple random sampling without replacement.

2. THE RATIO ESTIMATOR AND ITS LARGE SAMPLE BIAS AND MSE

For the sake of completeness and comparisons with our proposed estimators we briefly express the ratio estimator and its large sample bias and mean square error (MSE).

Let us assume that there are N units in the population under study and a without replacement simple random sample of n units is drawn from this population. Let (x_i, y_i), $i = 1,...n$, be the sample measurements on (x,y), y and x being respectively the main and auxiliary variables. Let $\bar{x} = (\Sigma_1^n x_i)/n$ and $\bar{y} = (\Sigma_1^n y_i)/n$ be the sample means for x and y respectively. Let $\bar{X}$ and $\bar{Y}$ denote the population mean for x and y respectively. Then the usual ratio estimator of $\bar{Y}$ is

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}} \bar{X} \tag{2.1}$$

Efficiency of a biased estimator is measured by the reciprocal of the amount of its mean square error (MSE). Thus the smaller the MSE the better the efficiency of the estimator. In many sample surveys reduction in MSE, even by a very small amount, plays an important role and increases efficiency significantly of the overall estimators. For example, in a stratified random sampling with L strata the MSE of the estimator of the population mean is approximately L times the MSE for the estimator of the individual stratum mean (Cochran 1977, p. 165). Thus, if L, the number of strate, is large, the overall MSE could be very large, and even a slight reduction in the MSE for the individual stratum mean estimator, would increase the efficiency of the overall population mean estimator significantly.

Since

$$\mathrm{Cov}(\overline{\tfrac{y}{x}},\bar{x}) = E(\bar{y}) - E(\overline{\tfrac{y}{x}})\,E(\bar{x})$$

$$= \bar{Y} - E(\bar{y}_R)$$

we have

$$E(\bar{y}_R) = \bar{Y} - \mathrm{cov}(\overline{\tfrac{y}{x}},\bar{x}),$$

so that the bias of the estimator $\bar{y}_R$ of $\bar{Y}$ is

$$B(\bar{y}_R) = ((\bar{y}_R) - \bar{Y}) = -\,\mathrm{cov}(\overline{\tfrac{y}{x}},\bar{x}) \quad .$$

When the sample size n is large so that the terms of the order $O(n^{-3/2})$ and lower can be ignored, the most commonly approximation to $MSE(\bar{y}_R)$ is

$$MSE(\bar{y}_R) = \frac{1-f}{n}\,[S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y], \qquad (2.2)$$

where $f = n/N$ is the sampling fraction, $R = \bar{Y}/\bar{X}$, and

$$S_y^2 = \frac{1}{N-1}\,\Sigma_1{}^N (y_i - \bar{Y})^2 \text{ and } S_x^2 = \frac{1}{N-1}\,\Sigma_1{}^N (x_i - \bar{X})^2$$

are population variances of y and x respectively, and ρ is the population correlation coefficient between x and y, defined by $\rho = \mathrm{cov}(x,y)/(S_x S_y)$.

3. SOME NEW RATIO TYPE ESTIMATORS AND THEIR EFFICIENCIES RELATIVE TO $\bar{y}_R$.

It is seen that the ratio estimator $\bar{y}_R = (\bar{y}/\bar{x})\bar{X}$ is dependent on $\bar{y}$ and $\bar{x}$, and, therefore, its efficiency is also dependent on the efficiencies of $\bar{y}$ and $\bar{x}$ as estimators of $\bar{Y}$ and $\bar{X}$ respectively.

As in Searl (1964), taking $\bar{y}^* = k\bar{y}$, for some constant k, as an estimator of $\bar{y}$, it can be seen that

$$\begin{aligned} MSE(\bar{y}^*) &= E(\bar{y}^* - \bar{Y})^2 = E(k\bar{y} - \bar{Y})^2 \\ &= k^2 \, var(\bar{y}) + (k-1)^2 \, \bar{Y}^2 \\ &= k^2 \left(\frac{1-f}{n}\right) S_y^2 + (k-1)^2 \, \bar{Y}^2 \end{aligned}$$

For the sake of simplicity in writing, we, throughout this paper, use the notation

$$\gamma = \frac{1-f}{n} = \frac{N-n}{Nn} .$$

The above expression for $MSE(\bar{y}^*)$ is minimized at $k = k_1$ given by

$$k_1 = [1 + \gamma \, C_y^2]^{-1} , \tag{3.1}$$

where

$$C_y = \frac{S_y}{\bar{Y}}$$

is the coefficient of variation for y. With this choice of k, the minimum MSE of

$$\bar{y}^* = [1 + \gamma \, C_y^2]^{-1} \, \bar{y} , \tag{3.2}$$

as an estimator of $\bar{Y}$ is given by

$$MSE(\bar{y}^*) = [1 + \gamma \, C_y^2]^{-1} \, \gamma \, S_y^2 \tag{3.3}$$

which is clearly less than

$$MS(\bar{y}) = var(\bar{y}) = \gamma\ S_y^2\ , \tag{3.4}$$

the mean square error of the mean per unit estimator $\bar{y}$.

Thus, when we have a prior knowledge of C_y, the coefficient of variation of y, we can construct a more efficient estimator $\bar{y}^*$ than $\bar{y}$ for estimating the population mean $\bar{Y}$. A prior knowledge of C_y sometime can be obtained from a most recent survey taken in the past or by conducting a preliminary survey by utilizing a small fraction of the budget allocated for the present survey. Thus it is not entirely unrealistic to assume a prior knowledge of C_y.

Assuming that we have a prior knowledge of C_y, we propose a ratio type estimator

$$\hat{R}_1 = \frac{\bar{y}^*}{\bar{x}} \tag{3.5}$$

for the ratio

$$R = \frac{Y}{X} = \frac{\text{Population Total for y}}{\text{Population Total for x}}\ ,$$

and a ratio type estimator

$$\bar{y}_1 = \frac{\bar{y}^*}{\bar{x}}\cdot\bar{X} \tag{3.6}$$

for $\bar{Y}$, where $\bar{y}^*$ is given by (2.4).

Now we compute $MSE(\hat{R}_1)$ and $MSE(\bar{y}_1)$. Since $\bar{y}_1 = \hat{R}_1\bar{X}$, to compute $MSE(\bar{y}_1)$ it suffices to compute $MSE(\hat{R}_1)$.

$$MSE(\hat{R}_1) = E(\hat{R}_1 - R)^2$$

$$= E\left(\frac{\bar{y}^* - R\bar{x}}{\bar{x}}\right)^2 = \frac{1}{\bar{X}^2}\ E\left\{\frac{(\bar{y}^* - R\bar{x}}{(1+\frac{\bar{x}-\bar{X}}{\bar{X}})}\right\}^2$$

$$= \frac{1}{\bar{X}^2} E\{(\bar{y}^* - R\bar{x})^2 \, (1+(\frac{\bar{x}-\bar{X}}{\bar{X}}))^{-2}\}$$

$$= \frac{1}{\bar{X}^2} E\{(\bar{y}^* - R\bar{x})^2 \, (1-2\,\frac{\bar{x}-\bar{X}}{\bar{X}} + 3\,(\frac{\bar{x}-\bar{X}}{\bar{X}})^2 \,\ldots)$$

Retaining the first term which is of order n^{-1} and ignoring the terms of lower order, we have the large sample approximation to $MSE(\hat{R}_1)$ as

$$MSE(\hat{R}_1) = \frac{1}{\bar{X}^2} E(\bar{y}^* - R\bar{x})^2 \tag{3.7}$$

$$= \frac{1}{\bar{X}^2} E\{(\bar{y}^* - \bar{Y}) - R(\bar{x}-\bar{X})\}^2$$

$$= \frac{1}{\bar{X}^2}[E(\bar{y}^*-\bar{Y})^2 + R^2E(\bar{x}-\bar{X})^2 - 2Rk_1 \, cov(\bar{x},\bar{y})] \, ,$$

since $\bar{y}^* = k_1\bar{y}$ and $E(\bar{x}-\bar{X})(\bar{y}^*-\bar{Y}) = k_1 \, cov(\bar{x},\bar{y})$, where k_1 is as given in (2.1). Thus, since from Theorem 2.3, p. 25 of Cochran (1977), $cov(\bar{x},\bar{y}) = \gamma \, cov(x,y)$, from (3.3) and (3.7) we have

$$MSE(\hat{R}_1) = \frac{\gamma}{\bar{X}^2} \, [k_1S_y^2 + R^2S_x^2 - 2Rk_1\rho S_xS_y]_2 \tag{3.8}$$

From (3.8), since $MSE(\bar{y}_1) = \bar{X}^2 \, MSE(\hat{R}_1)$, we have

$$MSE(\bar{y}_1) = \gamma[k_1S_y^2 + R^2S_x^2 - 2Rk_1\rho S_xS_y]$$

$$= \gamma[(1-2\rho\frac{C_x}{C_y})k_1S_y^2 + R^2S_x^2] \tag{3.9}$$

Notice that $MSE(\bar{y}_R)$ in (2.2) can be rewritten as

$$MSE(\bar{y}_R) = \gamma[(1- \frac{2\rho C_n}{C_y})S_y^2 + R^2S_x^2] \tag{2.2$'$}$$

Thus $MSE(\bar{y}_1)$ is clearly smaller than $MSE(\bar{y}_R)$ whenever $\rho >$ $(C_y/2C_x)$.

We have thus proved the following theorem

Theorem 1. The mean square error of the estimator $\bar{y}_1$ in (3.6) for the population mean $\bar{Y}$ is given by

$$MSE(\bar{y}_1) = \frac{1-f}{n}[(1-2\rho\frac{C_x}{C_y})(1+\gamma C_y^2)^{-1}S_y^2 + R^2S_x^2] \qquad (3.10)$$

$$= \frac{1-f}{n}(\bar{Y}^2)[(1-2\rho\frac{C_x}{C_y})(1+\gamma C_y^2)^{-1}C_y^2 + C_x^2] \ ,$$

and if

$$\frac{1}{2}\frac{C_y}{C_x} > \rho \ , \qquad (3.11)$$

then

$$MSE(\bar{y}_1) < MSE(\bar{y}_R) \ . \qquad (3.12)$$

It is well known that (e.g. see Cochran 1977), that if $\rho > (C_x/2C_y)$, then $MSE(\bar{y}_R) < MSE(\bar{y}) = var(\bar{y})$. Thus, we get the following corollary to Theorem 1.

Corollary 1. If

$$(C_x/2C_y) < \rho < (C_y/2C_x), \qquad (3.13)$$

then

$$MSE(\bar{y}_1) < MSE(\bar{y}_R) < MSE(\bar{y}) \qquad (3.14)$$

Instead of considering a specific estimator $\bar{y}_1 = [1 + C_y^2(1-f)/n]^{-1}(\bar{y}/\bar{x})\bar{X}$ for $\bar{Y}$, let us consider a class of ratio type estimators

$$\bar{y}_k = \frac{k\bar{y}}{\bar{x}} \cdot \bar{X} \qquad (3.15)$$

for estimating the population mean $\bar{Y}$. Then taking

$$\hat{R}_k = \frac{k\bar{y}}{\bar{x}} \qquad (3.16)$$

as an estimator of the ratio R of the population mean of y to that of x, we can write $MSE(\hat{R}_k)$ as

$$MSE(\hat{R}_k) = E(\hat{R}_k-R)^2$$

$$= E\left[\frac{(k\bar{y} - R\bar{x})^2}{\bar{X}(1+((\bar{x}-\bar{X})/\bar{X}))}\right]$$

$$= \frac{1}{\bar{X}^2} E[(k\bar{y} - R\bar{x})^2(1-2(\frac{\bar{x}-\bar{X}}{\bar{X}}) + 3(\frac{\bar{x}-\bar{X}}{\bar{X}})^2 \ldots]$$

Again retaining the terms up to the order n^{-1} and ignoring the terms of lower order, we see that a large sample approximation to $MSE(\hat{R}_k)$ is given by

$$MSE(\hat{R}_k) = \frac{1}{\bar{X}^2} E(k\bar{y}-R\bar{x})^2$$

$$= \frac{1}{\bar{X}^2} E\{k(\bar{y}-\bar{Y}) - R(\bar{x}-\bar{X}) + (k-1)\bar{Y}\}^2$$

$$= \frac{1}{\bar{X}^2} [k^2E(\bar{y}-\bar{Y})^2 + (k-1)^2\bar{Y}^2 + R^2E(\bar{x}-\bar{X})^2 - 2Rk\,cov(\bar{x},\bar{y})]$$

since other product terms are equal to zero. Thus, $MSE(\bar{y}_k) = \bar{X}^2\, MSE(\hat{R}_k)$ is given by

$$MSE(\bar{y}_k) = \gamma[k^2S_y^2 + R^2S_x^2 - 2Rk\rho S_xS_y] + (k-1)^2\bar{Y}^2 \qquad (3.17)$$

where $\gamma = (1-f)/n$.

We thus proved the following theorem.

Theorem 2. The large sample approximation up to order $1/n$ to the mean square error of the estimator $\bar{y}_k$ of $\bar{Y}$ is given by (3.17). Further, if $\rho > 0$, then

$$MSE(\bar{y}_k) < MSE(\bar{y}_R) \qquad (3.18)$$

for every

$$0 < k < \frac{2\gamma\rho C_xC_y+1-\gamma C_y^2}{1+\gamma C_y^2} \qquad (3.19)$$

where $\gamma = (1-f)/n$.

It can be checked easily that $MSE(\bar{y}_k)$ obtained in (3.17) achieves its minimum value for

$$k = k_2 = \frac{1+\gamma\rho C_x C_y}{1+\gamma C_y^2} \tag{3.20}$$

Taking this value in (3.20) of k, define a new ratio type estimator

$$\bar{y}_2 = \frac{k_2 \bar{y}}{\bar{x}} \cdot \bar{X} = \left(\frac{1+\gamma\rho C_x C_y}{1+\gamma C_y^2}\right) \frac{\bar{y}}{\bar{x}} \cdot \bar{X} \tag{3.21}$$

for the population mean $\bar{Y}$, where k_2 is as given in (3.20). Then by substituting the value (3.20) of k in (3.17) and simplifying the resulting expressions, it can be seen that the minimum value of MSE($\bar{y}_k$) is obtained as

$$MSE(\bar{y}_2) = \gamma\left[\frac{1-\rho^2\gamma C_x^2}{1+\gamma C_y^2} S_y^2 + R^2 S_x^2 - \frac{2\rho R S_x S_y}{1+\gamma C_y^2}\right] \tag{3.22}$$

$$= \gamma\left[\frac{1-\rho^2\gamma C_x^2 - 2\rho(C_x/C_y)}{1+\gamma C_y^2} S_y^2 + R^2 S_x^2\right] \tag{3.23}$$

where $\gamma = (1-f)/n$.

<u>Theorem 3</u>. <u>If</u> $\bar{y}_2$ <u>given in</u> (2.21) <u>is taken as an estimator of</u> $\bar{Y}$, <u>then its large sample mean square error is given by</u> (2.22) <u>or</u> (2.23). <u>Further, from</u> (3.22) <u>and</u> (3.23),

$$MSE(\bar{y}_2) < MSE(\bar{y}_1) \tag{3.24}$$

<u>for all values of</u> ρ, C_x <u>and</u> C_y.

After establishing that $\bar{y}_2$, as an estimator of $\bar{Y}$, is more efficient than $\bar{y}_1$, without any condition on the values of ρ, C_x and C_y, it is interesting to investigate how $\bar{y}_2$ performs as an estimator of $\bar{Y}$ compared to the usual ratio estimator. In this regard we have the following theorem,

<u>Theorem 4. The ratio type estimator $\bar{y}_2$ is always more efficient than the usual ratio estimator $\bar{y}_R$ for estimating the population mean $\bar{Y}$; that is</u>

$$\text{MSE}(\bar{y}_2) < \text{MSE}(\bar{y}_R) \tag{3.25}$$

<u>regardless the values of ρ, C_x and C_y.</u>

<u>Proof</u>: Notice that from (2.2), $\text{MSE}(\bar{y}_R)$ can be rewritten as

$$\text{MSE}(\bar{y}_R) = \gamma\left[(1-\frac{2\rho C_x}{C_y})S_y^2 + R^2 S_x^2\right]$$

Thus, from (3.23), $\text{MSE}(\bar{y}_2) < \text{MSE}(\bar{y}_R)$ if and only if

$$[1-\rho^2\gamma C_x^2 - 2\rho(C_x/C_y)] < [1-2\rho(C_x/C_y)](1+\gamma C_y^2)$$

or, if and only if

$$\rho^2\gamma C_x^2 > [1-2\rho(C_x/C_y)](-\gamma C_y^2)$$

or, if and only if

$$\rho^2(C_x^2/C_y^2) + 1-2\rho(C_x/C_y) > 0$$

which is always true, since the left hand side is $[1-\rho(C_x/C_y)]^2$.

Not only that $\bar{y}_2$ is more efficient that $\bar{y}_R$, we can compute the exact reduction in MSE due to use of $\bar{y}_2$ instead of $\bar{y}_R$ for estimating $\bar{Y}$. In this context we have the following theorem

<u>Theorem 5. The relation</u>

$$\text{MSE}(\bar{y}_R) = \text{MSE}(\bar{y}_2) + \frac{[1-\rho(C_x/C_y)]^2}{1+\gamma C_y^2} \cdot \gamma^2 C_y^2 S_y^2 \tag{3.26}$$

<u>always holds</u>.

The following theorem gives the exact reduction in MSE due to use of $\bar{y}_2$ instead of $\bar{y}_1$ as an estimator of $\bar{Y}$.

Theorem 6. The relation

$$MSE(\bar{y}_1) = MSE(\bar{y}_2) + \frac{(\gamma\rho C_x S_y)^2}{1+\gamma C_y^2} \tag{3.27}$$

always holds.

Further, since $MSE(\bar{y}_R)$ can be written as $\gamma[1-2\rho(C_x/C_y)]S_y^2 + R^2S_x^2]$, it follows that from (3.9) that

$$MSE(\bar{y}_R) = MSE(\bar{y}_1) + \frac{\gamma^2 C_y^2}{1+\gamma C_y^2}(1-R\rho(C_x/C_y))S_y^2 \tag{3.28}$$

which is positive for $\rho < C_y/(2C_x)$. Finally, as it is well known, the variance $(\bar{y}) = MSE(\bar{y})$ of the usual simple random sample mean, $\bar{y}$ taken as an estimator of the population mean $\bar{Y}$, can be split into two parts as

$$var(\bar{y}) = MSE(\bar{y}) = \gamma S_y^2 = MSE(\bar{y}_R) + (2\rho C_x C_y - C_x^2)\gamma S_y^2 \bar{Y}^2 \ ,$$

the reduction in MSE by use of $\bar{y}_R$ instead of $\bar{y}$ as an estimator of $\bar{Y}$ is $(2\rho C_x C_y - C_x^2)\gamma\bar{Y}^2 S_y^2$, which is positive for $\rho > C_x/(2C_y)$.

4. CONCLUDING REMARKS

In conclusion $\bar{y}_2$ is always preferable to $\bar{y}_1$, which in turn is preferable to $\bar{y}_R$ if $0 < \rho < C_y/(2C_x)$; and the latter in turn is preferable to mean per unit estimator $\bar{y}$ if $\rho > C_x/(2C_y)$. This concludes that if $(C_x/(2C_y)) < \rho < (C_y/2C_x)$, then $\bar{y}_2$, as an estimator of $\bar{Y}$, is more efficient than any of the estimators $\bar{y}$, $\bar{y}_R$ or $\bar{y}_1$. In fact, in the case $(C_x/2C_y) < \rho < (C_y/2C_x)$, we have the relation

$$MSE(\bar{y}_2) < MSE(\bar{y}_1) < MSE(\bar{y}_R) < MSE(\bar{y}) \ .$$

It may, however, be noted that construction of $\bar{y}_1$ requires a prior knowledge of the coefficient of variation C_y, and the construction of $\bar{y}_2$ requires a prior knowledge of ρ, C_x and C_y. As noted earlier, in practical sample surveys, a prior value of ρ, C_x or C_y can be guessed by utilizing appropriate information from a most recent survey taken in the past or by conducting a preliminary survey utilizing a small fraction of the full budget allocated for the current survey.

For construction of confidence intervals for the population mean $\bar{Y}$ based on our ratio type estimators $\bar{y}_1$ and $\bar{y}_2$, it is necessary that we estimate $MSE(\bar{y}_1)$ and $MSE(\bar{y}_2)$. Since sample variances s_x^2 and s_y^2, given by

$$s_x^2 = \frac{1}{n-1} \Sigma_1^n (x_i - \bar{x})^2$$

and

$$s_y^2 = \frac{1}{n-1} \Sigma_1^n (y_i - \bar{y})^2$$

are unbiased estimators for S_x^2 and S_y^2 respectively, we can estimate C_x and C_y respectively by $\hat{C}_x = (s_x/\bar{x})$ and $\hat{C}_y = (s_y/\bar{y})$. Similarly, we can estimate the population correlation coefficient ρ between x and y by the sample correlation coefficient

$$r = \frac{s_{xy}}{s_x s_y}, \text{ where } s_{xy} = \frac{1}{n-1} \Sigma_1^n (x_i - \bar{x})(y_i - \bar{y})$$

Now estimates of $MSE(\bar{y}_1)$ and $MSE(\bar{y}_2)$ can be obtained from (3.10) and (3.22) after substituting S_y^2, S_x^2, C_x, C_y and ρ respectively by s_y^2, s_x^2, $\hat{C}_x$, $\hat{C}_y$ and r.

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Received by Editorial Board member June 1987; Revised August 1988.

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